

Mathematics — Quick Revision Sheet

JEE Main & JEE Advanced

Formulas · Concepts · Exam Traps

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CONSTANTS & VALUES

$$\sqrt{2} = 1.414 \quad \sqrt{3} = 1.732 \quad \sqrt{5} = 2.236 \quad \pi = 3.1416 \quad e = 2.718 \quad \ln 2 = 0.693 \quad \ln 10 = 2.303 \quad \log_{10} 2 = 0.301 \quad \log_{10} 3 = 0.477$$

$$\log_{10} e = 0.434$$

Algebra

Sets, Relations & Functions

Set operations

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$(A \cup B)' = A' \cap B' \quad (A \cap B)' = A' \cup B'$$

Counting ($|A| = m, |B| = n$)

$$|P(A)| = 2^m \quad \text{relations} = 2^{mn} \quad \text{functions} = n^m$$

$$\text{one-one} = {}^n P_m$$

Onto functions

$$\sum_{k=0}^n (-1)^k {}^n C_k (n-k)^m$$

Periods

$$\sin ax: \frac{2\pi}{|a|} \quad \tan ax: \frac{\pi}{|a|}$$

Equivalence relation: reflexive, symmetric, transitive. Inverse exists iff bijective. $(f \circ g) \neq (g \circ f)$ generally. Even: $f(-x) = f(x)$; odd: $f(-x) = -f(x)$.

Complex Numbers

Forms

$$z = x + iy = r(\cos \theta + i \sin \theta) = re^{i\theta}$$

Modulus

$$|z| = \sqrt{x^2 + y^2} \quad z\bar{z} = |z|^2 \quad |z_1 z_2| = |z_1| |z_2|$$

Inequality

$$|z_1 + z_2| \leq |z_1| + |z_2| \quad ||z_1| - |z_2|| \leq |z_1 - z_2|$$

De Moivre

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

Roots of unity

$$z_k = e^{2\pi i k/n} \quad 1 + \omega + \omega^2 = 0, \omega^3 = 1 \quad \sum z_k = 0$$

Loci

$$|z - z_0| = r \quad (\text{circle}) \quad |z - z_1| = |z - z_2| \quad (\text{perp. bisector})$$

Rotation

$$\frac{z_2 - z_0}{z_1 - z_0} = \left| \frac{z_2 - z_0}{z_1 - z_0} \right| e^{i\theta}$$

$\arg(z_1 z_2) = \arg z_1 + \arg z_2; \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2; \omega = \frac{-1 \pm i\sqrt{3}}{2}$. Complex roots of real polynomials occur in conjugate pairs.

Quadratic Equations

Roots

$$x = \frac{-b \pm \sqrt{D}}{2a}, \quad D = b^2 - 4ac$$

Sum & product

$$\alpha + \beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{a} \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Cubic

$$\sum \alpha = -\frac{b}{a} \quad \sum \alpha\beta = \frac{c}{a} \quad \alpha\beta\gamma = -\frac{d}{a}$$

Common root

$$(c_1 a_2 - c_2 a_1)^2 = (a_1 b_2 - a_2 b_1)(b_1 c_2 - b_2 c_1)$$

Extreme value

$$x = -\frac{b}{2a} \quad y_{ext} = -\frac{D}{4a}$$

$D > 0$ real distinct; $D = 0$ equal; $D < 0$ complex. With rational coefficients, irrational/complex roots come in pairs.
 $ax^2 + bx + c > 0 \quad \forall x \Leftrightarrow a > 0, D < 0$.

Sequences & Series

Arithmetic progression

$$T_n = a + (n-1)d \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

Geometric progression

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Means

$$AM = \frac{a+b}{2} \quad GM = \sqrt{ab} \quad HM = \frac{2ab}{a+b} \quad AM \geq GM \geq HM$$

Standard sums

$$\sum n = \frac{n(n+1)}{2} \quad \sum n^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

AGP

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$GM^2 = AM \cdot HM$. Telescoping: $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$. HP: reciprocals form an AP. n AMs between a, b : $d = \frac{b-a}{n+1}$.

■ Permutations & Combinations

Basics

$${}^nP_r = \frac{n!}{(n-r)!} \quad {}^nC_r = \frac{n!}{r!(n-r)!} \quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

Circular

$$(n-1)! \quad \frac{(n-1)!}{2} \quad (\text{necklace})$$

Repeated objects

$$\frac{n!}{p!q!r!} \quad {}^{n+r-1}C_{r-1} \quad (\text{identical objects into } r \text{ boxes})$$

Derangements

$$D_n = n! \sum_{k=0}^n \frac{(-1)^k}{k!}$$

Divisors of $p^a q^b r^c$

$$(a+1)(b+1)(c+1)$$

Equal groups

$$\frac{(mn)!}{(n!)^m m!}$$

Diagonals of n -gon: $\frac{n(n-3)}{2}$. Selections of ≥ 1 from n distinct: $2^n - 1$. Use complement for "at least".

■ Binomial Theorem

Expansion

$$(x+y)^n = \sum_{r=0}^n {}^nC_r x^{n-r} y^r \quad T_{r+1} = {}^nC_r x^{n-r} y^r$$

Coefficient sums

$$\sum {}^nC_r = 2^n \quad \sum (-1)^r {}^nC_r = 0 \quad \sum r {}^nC_r = n2^{n-1}$$

Middle term

$$n \text{ even: } T_{\frac{n}{2}+1} \quad n \text{ odd: } T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$$

General index

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots \quad (|x| < 1)$$

Vandermonde

$$\sum_k {}^mC_k {}^nC_{r-k} = {}^{m+n}C_r$$

Greatest term of $(1+x)^n$: $T_{r+1} \geq T_r \Leftrightarrow r \leq \frac{(n+1)x}{1+x}$. Multinomial coefficient: $\frac{n!}{a!b!c!}$.

■ Matrices & Determinants

Inverse

$$A^{-1} = \frac{\text{adj}A}{|A|} \quad A \cdot \text{adj}A = |A| I$$

Properties

$$|kA| = k^n |A| \quad |AB| = |A| |B| \quad |\text{adj}A| = |A|^{n-1} \\ (AB)^T = B^T A^T \quad (AB)^{-1} = B^{-1} A^{-1}$$

Cayley-Hamilton (2×2)

$$A^2 - (\text{tr}A)A + |A|I = O$$

Cramer's rule

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}$$

Vandermonde

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$AX = B$: $|A| \neq 0$ unique solution; $|A| = 0$ and $(\text{adj}A)B = O$ infinite; else none. Symmetric $A^T = A$; skew $A^T = -A$ (odd order: $|A| = 0$). Orthogonal: $AA^T = I$.

Trigonometry

Trigonometric Ratios & Equations

Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \tan^2 \theta = \sec^2 \theta$$

Compound angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Multiple angles

$$\sin 2A = 2 \sin A \cos A \quad \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A \quad \cos 3A = 4 \cos^3 A - 3 \cos A$$

Sum to product

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

General solutions

$$\sin \theta = \sin \alpha: \theta = n\pi + (-1)^n \alpha \quad \cos \theta = \cos \alpha: \theta = 2n\pi \pm \alpha$$

$$\tan \theta = \tan \alpha: \theta = n\pi + \alpha$$

Range

$$-\sqrt{a^2 + b^2} \leq a \sin \theta + b \cos \theta \leq \sqrt{a^2 + b^2}$$

Standard values

$$\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4} \quad \sin 18^\circ = \frac{\sqrt{5} - 1}{4} \quad \cos 36^\circ = \frac{\sqrt{5} + 1}{4}$$

$$\tan 15^\circ = 2 - \sqrt{3}$$

Properties of Triangles

Sine & cosine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Area

$$\Delta = \frac{1}{2} bc \sin A = \sqrt{s(s-a)(s-b)(s-c)}$$

Radii

$$r = \frac{\Delta}{s} \quad R = \frac{abc}{4\Delta} \quad r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

Projection & half-angle

$$a = b \cos C + c \cos B \quad \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Heights & distances: angle of elevation = angle of depression (alternate angles). Sum of angles $A + B + C = \pi$;
 $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

Inverse Trigonometric Functions

Principal values

$$\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \cos^{-1} x \in [0, \pi] \quad \tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Relations

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

Sum formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \quad (xy < 1)$$

Double angle

$$2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$$

$\sin^{-1}(\sin x) = x$ only for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Always check the principal range before simplifying.

Coordinate Geometry

Straight Lines

Distance & section

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Line forms

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad \frac{x}{a} + \frac{y}{b} = 1$$

$$x \cos \alpha + y \sin \alpha = p$$

Angle between lines

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Distance from a point

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \quad \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} \quad (\text{parallel lines})$$

Area of triangle

$$\Delta = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Triangle centres

$$G = \left(\frac{\sum x}{3}, \frac{\sum y}{3} \right) \quad I = \frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

Family of lines: $L_1 + \lambda L_2 = 0$. Parallel: $m_1 = m_2$; perpendicular: $m_1 m_2 = -1$. Foot of perpendicular: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$ (double for the image).

Circles

Equations

$$(x-h)^2 + (y-k)^2 = r^2 \quad x^2 + y^2 + 2gx + 2fy + c = 0$$

$$r = \sqrt{g^2 + f^2 - c}$$

Tangent

$$T = 0: xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$\text{length} = \sqrt{S_1}$$

Chords

$$\text{chord of contact: } T = 0 \quad \text{midpoint chord: } T = S_1$$

Tangency condition

$$y = mx + c \text{ touches } x^2 + y^2 = a^2 \Leftrightarrow c^2 = a^2(1 + m^2)$$

Two circles

$$\text{radical axis: } S_1 - S_2 = 0 \quad \text{orthogonal: } 2g_1 g_2 + 2f_1 f_2 = c_1 + c_2$$

Common tangents: $d > r_1 + r_2$: 4; $d = r_1 + r_2$: 3; intersecting: 2; internal touch: 1; one inside other: 0. Chord length = $2\sqrt{r^2 - p^2}$.

■ Parabola, Ellipse & Hyperbola

Parabola $y^2 = 4ax$

focus $(a, 0)$ directrix $x = -a$ LR $= 4a$ $(at^2, 2at)$

Parabola lines

$$ty = x + at^2 \quad y = mx + \frac{a}{m} \quad y = mx - 2am - am^3$$

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad e = \sqrt{1 - \frac{b^2}{a^2}} \quad \text{foci } (\pm ae, 0) \quad \text{LR} = \frac{2b^2}{a}$$

Ellipse lines

$$y = mx \pm \sqrt{a^2m^2 + b^2} \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad PS + PS' = 2a$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad e = \sqrt{1 + \frac{b^2}{a^2}} \quad y = \pm \frac{b}{a}x \quad (\text{asymptotes})$$

Hyperbola lines

$$y = mx \pm \sqrt{a^2m^2 - b^2} \quad |PS - PS'| = 2a$$

$$xy = c^2: \left(ct, \frac{c}{t}\right), e = \sqrt{2}$$

Eccentricity: parabola $= 1$, ellipse < 1 , hyperbola > 1 . Director circle: ellipse $x^2 + y^2 = a^2 + b^2$; hyperbola $x^2 + y^2 = a^2 - b^2$. Auxiliary circle: $x^2 + y^2 = a^2$.

Calculus

■ Limits & Continuity

Standard limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$$

More limits

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1 \quad \lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k \quad \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

1^∞ form

$$\lim f^g = e^{\lim g(f-1)}$$

L'Hôpital

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)} \quad \left(\frac{0}{0}, \frac{\infty}{\infty}\right)$$

Continuity

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

Series

$$e^x = 1 + x + \frac{x^2}{2!} + \dots \quad \sin x = x - \frac{x^3}{3!} + \dots \quad \cos x = 1 - \frac{x^2}{2!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$. Differentiable $\Rightarrow$ continuous, not conversely ($|x|$ at 0). Squeeze theorem for bounded oscillation.

■ Differentiation

Standard derivatives

$$(x^n)' = nx^{n-1} \quad (e^x)' = e^x \quad (a^x)' = a^x \ln a \quad (\ln x)' = \frac{1}{x}$$

Trigonometric

$$(\sin x)' = \cos x \quad (\tan x)' = \sec^2 x \quad (\sec x)' = \sec x \tan x$$

$$(\cot x)' = -\csc^2 x$$

Inverse trigonometric

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}} \quad (\tan^{-1} x)' = \frac{1}{1+x^2}$$

$$(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

Rules

$$(uv)' = u'v + uv' \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Logarithmic differentiation

$$y = u^v \Rightarrow y' = u^v \left(v' \ln u + \frac{vu'}{u}\right)$$

Mean value theorems

$$\text{Rolle: } f'(c) = 0 \quad \text{LMVT: } f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$\frac{d}{dx} |x| = \frac{x}{|x|}. \text{ Leibniz: } (uv)^{(n)} = \sum {}^nC_r u^{(n-r)} v^{(r)}. \text{ Inverse: } \frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}.$$

■ Applications of Derivatives

Tangent & normal

$$m = \left. \frac{dy}{dx} \right|_{(x_1, y_1)} \quad y - y_1 = m(x - x_1) \quad m_{\text{normal}} = -\frac{1}{m}$$

Angle between curves

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Monotonicity

$$f' > 0 \Rightarrow \text{increasing} \quad f' < 0 \Rightarrow \text{decreasing}$$

Extrema

$$f'(c) = 0, f''(c) < 0 \Rightarrow \max \quad f''(c) > 0 \Rightarrow \min$$

Approximation

$$f(x + \Delta x) \approx f(x) + f'(x) \Delta x$$

Subtangent = $\left| \frac{y}{m} \right|$, subnormal = $|ym|$. Optimisation: reduce to one variable, set $f' = 0$, verify with f'' or sign change. Inflection: f'' changes sign.

■ Integration

Basic

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \int \frac{dx}{x} = \ln |x| \quad \int a^x dx = \frac{a^x}{\ln a}$$

$$\int \sec^2 x dx = \tan x$$

Trigonometric

$$\int \sec x dx = \ln |\sec x + \tan x| \quad \int \csc x dx = \ln \left| \tan \frac{x}{2} \right|$$

Algebraic forms

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

Root forms

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$

By parts

$$\int uv dx = u \int v dx - \int (u' \int v dx) dx$$

$$\int e^x [f(x) + f'(x)] dx = e^x f(x)$$

Definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx \quad \int_{-a}^a f = \begin{cases} 2 \int_0^a f & \text{even} \\ 0 & \text{odd} \end{cases}$$

Wallis

$$\int_0^{\pi/2} \sin^n x dx = \frac{(n-1)(n-3)\dots}{n(n-2)\dots} \times \left\{ \frac{\pi}{2} \quad (n \text{ even}) \right\}$$

Leibniz rule

$$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt = f(h) h' - f(g) g'$$

Sum as integral

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right) = \int_0^1 f(x) dx$$

Areas

$$A = \int_a^b (y_{\text{top}} - y_{\text{bottom}}) dx \quad \text{ellipse } \pi ab \quad \text{parabola-LR: } \frac{8a^2}{3}$$

between $y^2 = 4ax$, $x^2 = 4by$: $\frac{16ab}{3}$

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(2a-x) = f(x), \text{ else } 0 \text{ if } f(2a-x) = -f(x). \text{ Periodic: } \int_0^{nT} f = n \int_0^T f.$$

■ Differential Equations

Order & degree

order=highest derivative
degree=its power (after clearing radicals)

Variable separable

$$\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x) dx$$

Homogeneous

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right): \text{ put } y = vx$$

Linear first order

$$\frac{dy}{dx} + Py = Q \quad IF = e^{\int P dx} \quad y \cdot IF = \int Q \cdot IF dx$$

Bernoulli

$$\frac{dy}{dx} + Py = Qy^n: z = y^{1-n}$$

Growth-decay

$$\frac{dN}{dt} = kN \Rightarrow N = N_0 e^{kt}$$

Arbitrary constants in the general solution = order. Orthogonal trajectories: replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$. $\frac{dy}{dx} = f(ax + by + c)$: put $z = ax + by + c$.

Vectors, 3D & Probability

Vector Algebra

Products

$$\vec{a} \cdot \vec{b} = ab \cos \theta \quad \left| \vec{a} \times \vec{b} \right| = ab \sin \theta \quad \left| \vec{a} \times \vec{b} \right|^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$$

Triple products

$$\left[\vec{a} \vec{b} \vec{c} \right] = \vec{a} \cdot (\vec{b} \times \vec{c})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$$

Areas & volumes

$$\text{triangle} = \frac{1}{2} \left| \vec{a} \times \vec{b} \right| \quad \text{parallelepiped} = \left| \left[\vec{a} \vec{b} \vec{c} \right] \right|$$

$$\text{tetrahedron} = \frac{1}{6} \left| \left[\vec{a} \vec{b} \vec{c} \right] \right|$$

Section & projection

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n} \quad \text{proj}_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \vec{b}$$

$$\left[\vec{a} \vec{b} \vec{c} \right] = 0; \text{ collinear: } \vec{a} \times \vec{b} = \vec{0}.$$

$$\left| \vec{a} + \vec{b} \right|^2 = a^2 + b^2 + 2\vec{a} \cdot \vec{b}.$$

Three-Dimensional Geometry

Direction cosines

$$l^2 + m^2 + n^2 = 1 \quad \cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\sum a_i^2} \sqrt{\sum a_2^2}}$$

Line

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad \vec{r} = \vec{a} + \lambda \vec{b}$$

Plane

$$ax + by + cz + d = 0 \quad \vec{r} \cdot \vec{n} = d \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Distance point-plane

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Skew lines

$$d = \frac{\left| (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) \right|}{\left| \vec{b}_1 \times \vec{b}_2 \right|}$$

Angles

$$\sin \theta = \frac{\left| \vec{b} \cdot \vec{n} \right|}{\left| \vec{b} \right| \left| \vec{n} \right|} \left(\text{line-plane} \right) \quad \cos \theta = \frac{\left| \vec{n}_1 \cdot \vec{n}_2 \right|}{\left| \vec{n}_1 \right| \left| \vec{n}_2 \right|} \left(\text{planes} \right)$$

$$\text{Plane through intersection: } P_1 + \lambda P_2 = 0. \text{ Lines are coplanar iff } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0. \text{ Foot of perpendicular to plane:}$$

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = -\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}.$$

Probability

Basics

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A') = 1 - P(A)$$

Conditional

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{independent: } P(A \cap B) = P(A)P(B)$$

Total probability & Bayes

$$P(A) = \sum P(E_i)P(A|E_i)$$

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_j P(E_j)P(A|E_j)}$$

Binomial distribution

$$P(X=r) = {}^nC_r p^r q^{n-r} \quad \mu = np \quad \sigma^2 = npq$$

Random variable

$$E(X) = \sum x_i p_i \quad \text{Var}(X) = E(X^2) - [E(X)]^2$$

At least one success: $1 - q^n$. Mutually exclusive ($P(A \cap B) = 0$) is not the same as independent. Geometric: $P(X=k) = q^{k-1}p$.

Statistics

Central tendency

$$\bar{x} = \frac{\sum x_i}{n} \quad \text{Mode} \approx 3 \text{ Median} - 2 \text{ Mean}$$

Dispersion

$$\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 \quad MD = \frac{\sum |x_i - \bar{x}|}{n} \quad CV = \frac{\sigma}{\bar{x}} \times 100$$

Change of data

$$x \rightarrow x+k: \bar{x} \rightarrow \bar{x}+k, \sigma \text{ same} \quad x \rightarrow kx: k\bar{x}, |k| \sigma$$

Combined data

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \quad \sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

First n natural numbers: $\sigma^2 = \frac{n^2-1}{12}$; AP of common difference d : $\sigma^2 = \frac{d^2(n^2-1)}{12}$. Lower CV means more consistent data.

Mathematical Reasoning & Miscellaneous

Logic

$$\sim(p \wedge q) \equiv \sim p \vee \sim q \quad \sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$p \Rightarrow q \equiv \sim q \Rightarrow \sim p \quad p \Rightarrow q \equiv \sim p \vee q$$

Negation

$$\sim(p \Rightarrow q) \equiv p \wedge \sim q$$

Logarithms

$$\log_a(xy) = \log_a x + \log_a y \quad \log_a b = \frac{\log b}{\log a} \quad a^{\log_a b} = b$$

Modulus

$$|x| < a \Leftrightarrow -a < x < a \quad |x| > a \Leftrightarrow x < -a \text{ or } x > a$$

Greatest integer

$$[x] \leq x < [x] + 1 \quad \{x\} = x - [x]$$

A tautology is always true; a contradiction always false. "For all" negates to "there exists".